**Coin**

<https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:prob-comb/x9e81a4f98389efdf:prob-combinatorics-precalc/e/probability_with_perm_comb>

1. We toss a fair coin twice. What is the probability that: (a) We get heads then tails. (b) We get two tails. (c) The first toss turns up heads. (d) We get two heads. (e) We get exactly one head.
   * A) ¼ b) ¼ c) ½ d) ½ e) ¼
2. **If you flip a fair coin**4**times, what is the probability that you will get exactly**2**tails?**
   * **Binomial Distribution: nCr \* (P)^r \* (1-P)^(n-r)**
   * **4C2 \* (0.5)^2 \* (1-0.5)^(4-2) = 0.375**

**OR**

* + **There are 6 possible arrangements-- There are 4! Total arrangements of 4 tails, divided by 2! Of the order of “T” (tails), and 2! Of the order of “H” (heads) = 4!/2!2! = 6.**
  + **possible outcomes: Each flip has 2 options, do it 4 times, 2 \*4 = 16**
  + **6/16 = 0.375.**

**OR**

* **Write out and see that there are 6 valid combinations.**
* **Probability of getting 2 tails in any particular order is (1/2)^4.**
* **(1/2)^4 \* 6 = 0.375**

1. **Unfair Coin:** Probability of tails is 0.7. We toss the coin twice. Find the probability of the following events: (a) We get no heads. (b) We get exactly one head. (c) We get two heads.
   * a) TT = 0.7 \*0.7 = 0.49. b) (binomial distribution) 2C1 \* 0.3^1 \* 0.7 ^1 = 0.42. c) 0.3^2 = 0.09
2. Unfair Coin: An unbalanced coin comes up with tails 60% of the times its tossed. It is tossed 3 times in a row. What is the probability that all three tosses came up the same?
   * .0.6^3  + 0.4^3 = 0.28000
3. You have 4 coins: 3 coins are fair and one is a fake coin with heads on both sides. You pick one coin at random and toss it. What is the probability the toss comes up heads?
   * (3/4)(1/2) + (¼)(1)(1) = 0.625

**Dice**

1. A fair six-sided die is rolled twice. What is the probability of getting 1 on the ﬁrst roll and not getting 6 on the second roll?
2. Three dice are thrown. What is the probability of obtaining the same number on all three dice?

(1/6^3) \* 6 = 1/36

* + Since there are six possibilities as to what the triplet could be, we multiply by 6. If we wanted a specific number though, it would be (1/6^3).

1. A fair six-sided die is rolled 6 times. What is the probability that no two results are the same number?
   * 6/6 \* ⅚ \* 4/6 \* 3/6 \* 2/6 \* ⅙ = 0.0154

OR

* + 6! / 6^6 = 0.0154

1. A pair of fair six-sided dice is rolled. What is the probability that the sum of the dice is 9 or more?
   * All possibilities: (6,6), (6,5), (6,4), (6,3)   (5,6), (5,5), (5,4),   (4,6), (4,5),   (3,6) = 10
   * 10 / 36  = 0.27778

**Cards**

1. In a regular deck of cards, there are 52 cards from 4 suits of 13 cards each: Spades, Hearts, Diamonds and Clubs. You draw a card from the deck, note down its suit, return it to the deck and reshuﬄe. You repeat this 5 times. What is the probability not a single Diamonds card was drawn?
   * ¾ \* 5
2. You draw a card at random, what is the probability of it being a number card of value between 2-5 (inclusive)?

54 cards, choose one. Cards 2-5 totals to 16, choosing one. 16C1/54C1 = 0.2962

1. You draw two cards one after another, returning and reshuﬄing after each draw, what is the probability they are both Diamonds?

We play the game twice, so its 54 choose one twice. Each time, we have 13 possible cards.

(13C1)^2  / (54C1)^2  =  0.0579

1. You draw four cards one after another (without returning cards to the deck between draws). What is the probability the cards drawn included one of each of the four suits?

54 cards, choosing 4 cards. We want one of each suit, so it is 13 choose 1, all 4 times.

(13C1)^2  / 54C4 = .0903

1. You play a game where all number cards (2-10) have a value equal to their number, ace has a value of 1, face cards have a value of 10 and joker can have the same value as any card of your choice. To play the game, you draw three cards into your hand, and you win if your hand total (sum of values) meets or exceeds a stated goal. If the goal number is 4, what is your probability of winning in your ﬁrst draw?

To calculate, we will find the probability of *not* winning. The only possibility of not winning is getting all 1’s. True, jokers can be added to the possibilities, however the actual goal is to win, therefore any time a joker is drawn, it will be a high number instead. So we are only left with the possibility of losing by drawing all ones.

4c3 / 54c3

OR

1 - ((4/54)(3/53)(2/52)) = 0.99984

**Word combinations**

1. **How many unique ways to write PRIOR?**
   * **5 letters, so 5!**
   * **However, the two R’s are not unique. Therefore, we divide the combination by 2! -- 5!/2! = 60**
2. How many unique ways to write Tennessee?
   * 9! Total ways.
   * However, there is double S, double N, and 4 E’s. Therefore, 9!/2!2!4! = 3780.

**Orders in a line**

1. How many ways to arrange 4 people, but two of them need to sit next to each other?
   * Since two must sit together, we treat them as one entity, therefore total combinations = 3! = 6.
   * However, since the two that sit next to each other can be to the right of or the left of the other, we double the possibilities by two (A being on the left, and A being on the right, in each of the 6 orders)—6 \* 2 = 12.

OR

* Total combinations = 4!.
* 4!/2! = 12.

1. An illiterate person arranges the thirty-two volumes of the Encyclopedia Brittanica on the shelf at random. What is the probability that he will arrange the volumes in the correct order?
   * 1/32!
2. The Moskowitz family consists of the father, the mother, and five children. In how many different ways can the family be seated in a row of seven seats such that …
   * (a) the parents are sitting on the ends?
     1. 5! (the kids in between) \* 2 (parents on either side.)
   * (b) the father is sitting to the right of the mother (not necessarily next to her)?
     1. 5! / 2 (remove all options with father on the left)
   * (c) the parents are sitting next to each other?
     1. 6! (the kids and a parent) /2! (can switch either parent around)
   * (d) Yehuda, Nechemiah and Moshe (three of the children), are sitting next to each other?
     1. 5! /3!

**Choose from a Group**

1. There are 10 students in a class: 3 boys and 7 girls. **If the teacher picks a group of 3 at random, what is the probability that everyone in the group is a girl?**
   * **Total combinations: 10 choose 3, nCr = 10!/7!3!**
   * Desired choice: 7 girls, choose 3, nCr = 7!/4!3!
   * Desired choice/Total combinations = 7/24 = 0.29167
2. A bag contains 5 balls: 4 black and 1 white. A ball is drawn from the bag and returned to it, then another ball is drawn. What is the probability both were of the same color?
   * ⅘ \* ⅘ + ⅕ \* ⅕ = 0.68
3. 6. An aquarium contains 5 blue, 7 yellow and 3 red fish. Three fish are chosen at random. What is the probability that one of each color was chosen?
   * Total Combinations: 15c3
   * Desired Outcome: 5\*7\*3 , 5\*7\*3/15c3 = 0.230769
4. In a basket there are 3 apples, 2 oranges and 5 grapefruits.
   * (a) Two fruits are selected together from the basket. What is the probability of selecting an orange and an apple?
     + 2\*5/10c2
   * (b) A fruit is selected from the basket, and, without replacing the selected fruit, a second fruit is chosen. What is the probability of selecting an orange and an apple (in no particular order)?
     + 2\*3/10c2
   * (c) A fruit is selected from the basket, and, after replacing the selected fruit, a second fruit is chosen. What is the probability of selecting an orange and an apple (in no particular order)?
     + (2/10 \*3/10) \* 2

**Choose from Groups**

1. A Class of 45 students is randomly split into 3 practice groups of the same size. A pair of friends in the class really like to work together and hope they won’t end up in diﬀerent groups. What are the chances they’re assigned to the same group?

* Assume the 1st friend was the first to be assigned to a group, there are 14 empty spots, the probability our 2nd friend is given one of these spots should be: 14 / 44

OR

* Counting the number of ways our two friends end up in the same group, that team could be chosen in 3 ways. The other 13 students shall be chosen in C(43, 13) ways and the rest of the students may be assigned in C(30, 15) \* C(15, 15) ways.
* So → 3 \* C(43, 13) / C(45, 15) = (3 \* 43 \* 42 ... 31 / 13! ) / (45 \* 44 \* 43 ... 31 / 15!) = 3 \* 15! / 13! \* 44 \* 45 = 14 / 44

1. A box of matches contains 50 matches, of which 10 have already been used. Three matches are taken out of the box. What is the probability that at least one has not been used?
   * Redefine the question to be, “What is the probability all are used?”, and subtract the problem by 1.
   * Total Combinations: 50c3
   * Desired Outcome: 10c3 , 1 – (10c3 / 50c3) = .99388
2. 21 students are divided into two groups, one with 10 students and the other with 11 students. What is the probability that Reuven, Shimon and Levi will be in the same group?
   * Total Combinations: 21c3
   * Desired Outcomes: 10c3 or 11c3 , (10c3 + 11c3) / 21c3 = .214
3. A certain computer company has five programmers, five salespeople, five systems analysts, and five administrators. Each year five employees are randomly selected to attend a training conference at a resort hotel.
   * (a) What is the probability that the five chosen have the same profession?
     + 4/20c5
   * (b) What is the probability that the five chosen include all four professions?
     + Total combinations: 20c5
     + Desired Outcome: For 3 positions, there are 5 candidates each. For one position, there are 5c2 (they are a pairing). Therefore, 5^3 \* (4\*5c2)
     + 5^3 \* (4\*5c2) / 20c5 =
   * (c) What is the probability that among the five chosen, there are exactly two salespeople?
     + 5c2 / 20c5
4. An exam contains 5 questions and 5 answers that must be matched. What is the probability that a student who does not know anything but chooses the answers randomly will match the right answers to at least three questions?
   * Total Combinations: 5!
   * Desired outcome: At least three answers right, out of 5 (3/5, 4/5, 5/5). This is 5c3, and getting 4/5 is the same as getting 5/5 correct, therefore this last possibility = 1.
   * (5c3 +1)/ 5! = 0.09167

**Pairs of Socks**

1. Reuven wants to take a pair of socks from the closet, but it turns out the socks are not arranged in pairs. There are 10 individual socks that belong to 5 different pairs.

(a) If he takes 3 socks at random, what is the probability of getting a pair?

* Total Possibilities: 10C3
* Desired outcome: 5C1 \* 8C1 (5 pairs, choose one pair, 8 socks left, choose any one).
* (5\*8) / 10C3 = .009259

**Math**

1. **How many numbers between 1 and 100 (inclusive) are divisible by 3 or 2?**
   * **Divisible by 3 : 100/3 = 33.**
   * **Divisible by 2: 100/2 = 50.**
   * **Divisible by both 3 and 2 (getting rid of double counting)—3\*2 = 6, 100/6 = 16.**
   * **83-16 =67.**
2. **How many numbers between 1 and 100 (inclusive) are divisible by 4 or 5?**
   * 100/4 + 100/5 - 100/(4\*5) == 40